As a matter of interest in calculating the axial stresses, if the temperatures at $\bar{x} = \bar{x}_{c\ it}$ are calculated by using the simple fin solution

$$T - T_\infty = -\frac{ql}{k}\frac{2\bar{a}}{1/K + [(1/K^2) + 2\bar{h}]^{1/2}} \times \exp\left\{\left[\frac{1}{K} - \left(\frac{1}{K^2} + 2\bar{h}\right)^{1/2}\right]\bar{x}\right\}$$

some error will arise due to heat transfer by convection in the region $-\infty < \bar{x} < \bar{x}_{\ it}$ Figure 3 shows the error incurred

Reference

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Drop Size from a Liquid Jet in a Longitudinal Electric Field

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A relationship between the drop sizes found by the breakage of an unstable liquid jet with and without the presence of a longitudinal electric field is developed Experimental results are in satisfactory agreement with the relation derived

Introduction

EXCEPT for the classic example of a liquid issuing from a nozzle to form a jet, investigations of the dynamic stability of jets have not given much information on the resulting drop size found when the jet breaks up because of instability In the classic example, as Plateau[1] first showed, the cause of instability is the surface tension which makes the cylindrical jet an unstable figure of equilibrium In this case, Rayleigh[2] finds that the most probable drop diameter d (i e the drop diameter formed at the mode of maximum instability) for a jet of radius R is given by $d = 4\ 508R$ However, little information is given on the resulting drop size for other cases of instability which include the presence of an electric field,[3, 4] a magnetic field,[5, 6] any motion of the jet,[5] or a surrounding fluid [5, 7] The purpose of this paper is to study analytically and experimentally the size of the drops formed from an unstable jet when a longitudinal electric field is present

Analytical Study

Since for all cases of instability the most probable drop size is formed at the mode of maximum instability, one begins by first finding the mode of maximum instability Generally, it is found by perturbation of the Lagrange differential equation for the amplitude a of the deformation of the jet The resulting perturbation equation, which gives the criterion for stability, is then maximized to give the mode of maximum instability

In the classic case of the liquid jet, by assuming $a = \text{const} \times e^{\pm pt}$, the perturbation equation obtained is

$$p^2 = [xI_0'(x)T/I_0(x)\rho R^3](x^2 - 1) \qquad (1)$$

Received January 27, 1964 This work has been supported by the American Petroleum Institute

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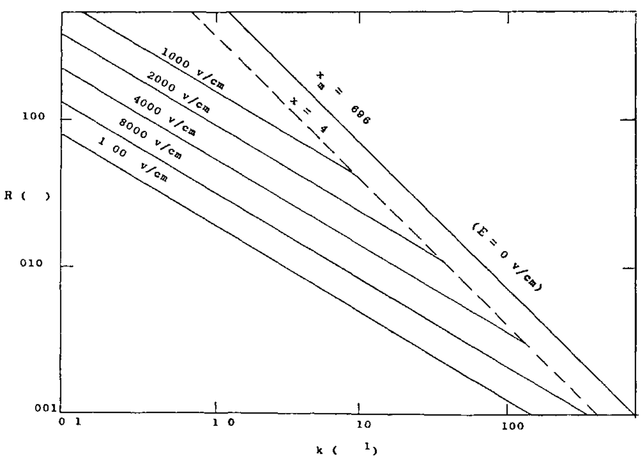

Fig 1 Relation between jet radius and wave number

Here R is the radius of the jet, T is the surface tension of the jet, ρ is the density of the jet, $x = Rk$, where k is the wave number, and I_0 is the zero-order Bessel function of the first kind for a pure imaginary argument The value x_m that causes p to be a maximum is given by Lamb[2] as $x_m = 0\ 696$ The plot of

$$x_m = kR = 0\ 696 \qquad (2)$$

for R vs k is shown in Fig 1

Following the foregoing procedure for the case of a jet in a longitudinal electric field **E**, Nayyar and Murty[4] obtain

$$p^2 = \frac{xI_1(x)}{\pi\rho R^2 I_0(x)} \times \left\{\frac{\pi T(1 - x^2)}{R} - \frac{(\epsilon_2 - \epsilon_1)^2 E^2 x I_0(x) K_0(x)}{4[\epsilon_1 I_1(x) K_0(x) + \epsilon_2 I_0(x) K_1(x)]}\right\} \qquad (3)$$

for the perturbation equation Here ϵ_1 is the dielectric constant of the jet, ϵ_2 is the dielectric constant of air, and K_n is the nth-order Bessel function of the second kind for an imaginary argument To find x_m, one considers $x \ll 1$ so that the Bessel functions can be replaced by their dominant terms One finds

$$x_m^2\left[-\gamma - \frac{1}{4} - \ln\frac{x_m}{2}\right] = \frac{2\pi\epsilon_2 T}{(\epsilon_2 - \epsilon_1)^2 E^2 R} \qquad (4)$$

Here γ is Euler's const $= 0\ 577$ For a jet issuing into

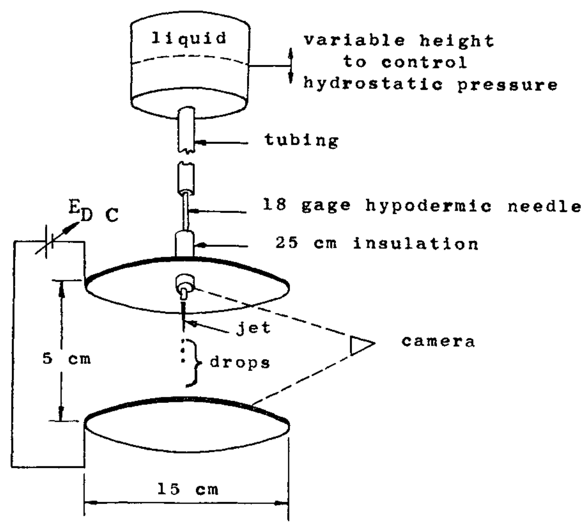

Fig 2 Experimental apparatus for producing and measuring d_i/d_E

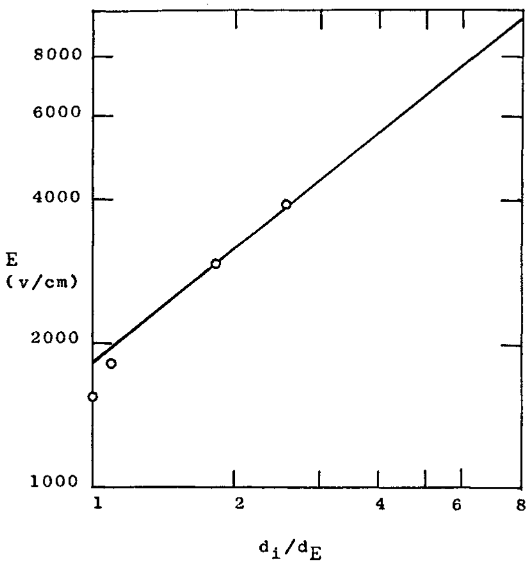

Fig 3 Comparison between predicted and experimental values of d_i/d_E for amyl alcohol

air, $\epsilon_2 = 1$ Hence, Eq (4) can be written as

$$x_m^2[-0\ 134 - \ln x_m] = 5\ 65(10)^5 A/E^2R \tag{5}$$

where $A = T/(1 - \epsilon_1)^2$ is dependent only upon the physical properties of the jet The plots of Eq (5) with $A = 0\ 01155$ (water) for R vs k parametric in **E** are essentially the straight lines shown in Fig 1 Here the plots of Eq (5) are terminated on the right (at $x_m = 0\ 4$) before intersecting the plot of Eq (2) This is done to indicate that Eq (5) is valid only when an electric field is present and for $x \ll 1$ However, if the exact relationship between R and k parametric in **E** were plotted in Fig 1, these lines would coincide with the existing plots of Eq (5) for $x \ll 1$, and then curve downward and become tangent to the plot of Eq (2) as

$$x_m \rightarrow 0\ 696 \qquad (\mathbf{E} \rightarrow 0) \tag{6}$$

It is noted in Fig 1 that, for a given decrease in the jet radius, the wave number in the electrical case k_E increases faster than the wave number in the nonelectrical case Since the slopes of Eqs (2) and (5) are constant when $\Delta \log R = \Delta \log R_E$, then

$$\frac{\Delta \log R/\Delta \log k}{\Delta \log R_E/\Delta \log k_E} = \frac{\Delta \log k_E}{\Delta \log k} = \frac{\log k_{f,E} - \log k_{i,E}}{\log k_f - \log k_i} = \text{const} \tag{7}$$

Here k_i is the wave number at the initial radius R_i, and k_f is the wave number at the final (reduced) radius R_f Since an applied electric field tends to stabilize the jet,[3–5] R_f is necessarily smaller than R_i Hence, the value of R_f can be determined from Fig 1 as the point of intersection of the vertical k_i line and the line of Eq (5) for the particular applied electric field

The convenient choice of $R_i = 1$ cm gives $k_i = 0\ 696$ cm^{-1} and $k_{i\ E} = 0\ 05$ cm^{-1} (for $\mathbf{E} = 1000$ v/cm); also $R_f = 0\ 1$ cm gives $k_f = 6\ 96$ cm^{-1} and $k_{f\ E} = 2\ 1$ cm^{-1} (for $\mathbf{E} =$ 1000 v/cm) Hence,

$$\text{const} = \frac{\log 2\ 1 - \log 0\ 05}{\log 6\ 96 - \log 0\ 696} = 1\ 623 \tag{8}$$

With this value, and the relationship $d_{f\ E} = \pi/k_{f\ E}$, Eq (7) becomes

$$d_{f\ E} = (\pi/k_{i\ E})(k_i/k_f)^{1\ 623} \tag{9}$$

For a jet with no electric field initially present, $R_{i\ E} = R_i$ so that $k_{i\ E} = k_i$, and the desired result is given by

$$d_E = \pi(k_i^{0\ 623}/k_f^{1\ 623}) \tag{10}$$

where $d_E = d_{f,E}$

Because of the approximation used in obtaining the mode of maximum instability given by Eq (5), some error can be expected at the lower electric field values Also, since the entire development of Eq (10) is based upon the mode of maximum instability, its accuracy will decrease as the mode of instability departs from the maximum mode In the experimental study that follows, the effect of both of these factors is clearly shown

Experimental Results

By using the relationship $d_i = \pi/k_i$, Eq (10) becomes

$$d_i/d_E = (k_f/k_i)^{1\ 623} \tag{11}$$

Since d_i and d_E can be produced and measured with the experimental arrangement shown in Fig 2, Eq (11) provides an experimental check of the analysis made

The solid curve in Fig 3 indicates the predicted values for amyl alcohol ($A = 0\ 1098$ dyne/cm) obtained from Eq (11) These values agree well with the experimental results, especially for $\mathbf{E} > 2000$ v/cm As expected, at lower values of **E**, the discrepancy between the experimental and theoretical results occurs from the approximation used in obtaining the mode of maximum instability given by Eq (5)

Conclusion

The preceding study of the drop size produced by a liquid jet in a longitudinal electric field yields an equation for the calculation of the drop size Experimental data indicate that this equation is accurate for a jet that is at the mode of maximum instability

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